Geometric Analysis of Sanxingdui Bronze Sun Wheel and Cold Mask Based on Mathematical Modeling and Image Processing

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ABSTRACT

The Sanxingdui site, located in southwest China and dating back 3000 - 5000 years, is a crucial origin of Chinese civilization. Its unearthed artifacts hold great historical, scientific, cultural, artistic, and aesthetic value. This study focuses on analyzing two significant artifacts - the bronze sun wheel and the golden mask - through mathematical modeling and image processing techniques. Firstly, for the bronze sun wheel: Geometric analysis of the bronze sun wheel. It has a diameter of 85 cm. To study its geometric features, we take the center as the coordinate origin. Using Matplotlib, a mathematical model is established. The curve equations of the sun wheel and its internal rays are derived, and patterns with different numbers of rays are generated. Secondly, for the golden mask: Geometric analysis and mass calculation of the golden mask. The golden mask is approximately 23 cm wide and 28 cm high, with only half of it having been found. To analyze its features and mass, we restore the half mask to a complete mask using symmetry recovery techniques. Taking the tip of the nose as the coordinate origin, we calculate the curve equations of the "double-eye-edges" and "double-ear-edges". Image processing tools are used to detect and extract the edges and contours, and feature points are fitted using spline interpolation to create smooth curves. The mask has a uniform thickness of 2 mm and is composed of 85% gold, 13% silver, and 2% impurities, with a density of about 18.36 g/cm^3. The fitted curves are then used to calculate the surface area and volume. The surface area and mass are determined to be 2468.432 square cm and 539.865 grams respectively. These calculations provide valuable data for understanding the geometric structure and manufacturing process of the golden mask, which supports archaeological research and cultural heritage protection.

Keywords: Geometric Features; Mass Calculation; Symmetry Recovery Techniques; Smooth Curves

1 INTRODUCTION

The Sanxingdui site, located in southwest China, has great historical and cultural significance. Dating back 3,000 to 5,000 years, it is the largest, longest - lasting, and most culturally rich ancient city, kingdom, and Shu culture site discovered so far. Hailed as one of the greatest archaeological discoveries of the 20th century, Sanxingdui shows that the Yangtze River basin, like the Yellow River basin, is a birthplace of Chinese civilization, thus earning the title "source of Yangtze civilization." This discovery reshapes the traditional understanding of the origins of Chinese civilization and also provides new perspectives for studying ancient Chinese cultures [1].

The unearthed artifacts from Sanxingdui are invaluable human cultural heritage, with great historical, scientific, cultural, artistic, and aesthetic value. There are unique treasures among them, such as the 2.62 - meter - tall bronze standing figure, the 1.38 - meter - wide bronze mask, the 3.95 - meter - tall bronze sacred tree, gold scepters, and jade artifacts. These artifacts not only show the unique charm and high - level craftsmanship of the ancient Shu civilization but also reflect the religious beliefs, lifestyle, and artistic achievements of that time. Each artifact contains rich historical information and is an important window for understanding the ancient Shu culture [2-4].

This study aims to explore the historical and cultural significance behind the bronze sun wheel and the golden mask unearthed from Sanxingdui through geometric analysis using mathematical modeling methods.

For the bronze sun wheel, first, take the center as the coordinate origin to calculate and draw sun wheel patterns with different numbers of rays. Specifically, a mathematical model will be established to derive the curve equations of the bronze sun wheel and its internal rays, and further analyze their geometric features. By drawing sun wheel patterns with four, six, eight, and twelve rays, we can gain a deeper understanding of the design concept and symbolic meaning of this artifact [5].

For the golden mask, it needs to be restored to its complete form, and its geometric features, estimated surface area, and mass calculated. Using symmetry recovery techniques, the half - mask will be restored to a complete mask. Then, taking the tip of the nose as the coordinate origin, a coordinate system will be established to calculate the curve equations of the double - eye edges and double - ear edges. Finally, surface area estimation and mass calculation will be carried out through surface integration methods, combined with the density of gold to estimate its mass. These geometric analyses not only reveal the manufacturing process and design ideas of the golden mask but also provide valuable data support for archaeological research [6].

These studies further reveal the unique charm and scientific value of Sanxingdui artifacts. The analyses not only help to understand the achievements of the ancient Shu civilization but also provide a scientific basis for protecting and preserving these invaluable cultural heritages. Moreover, these research results will serve as references for future archaeological research and cultural heritage protection.

2 RELATED WORK AND ASSUMPTIONS

2.1 Related work

Work one: To analyze the bronze sun wheel, a rectangular coordinate system will be established with the center of the wheel as the origin. The outer contour of the sun wheel can be considered a circle, and its standard circular equation needs to be derived. Based on the work's description, the equations of the five internal arcs (rays) will be solved. These arc equations will be used to derive a general equation for the arcs, which can be expressed parametrically or in polar coordinates [7].

Steps to solve work One:

(1) Establish Coordinate System: Place the center of the bronze sun wheel at the origin of the coordinate system.

(2) Outer Contour Equation: Derive the standard equation of the circle representing the outer contour of the sun wheel.

(3) Internal Arcs: Solve for the equations of the five internal arcs (rays) and derive the general equation for these arcs.

(4) Geometric Model: Use the derived curve equations to build the geometric model of the bronze sun wheel.

(5) Sun Wheel Patterns: Adjust the geometric model based on the number of rays (four, six, eight, twelve) to draw the corresponding sun wheel patterns.

Work two: For the golden mask from the Sanxingdui site, which is approximately 23 cm wide and 28 cm high, only half of the mask has been found. To estimate the surface area and mass of the complete mask, symmetry recovery techniques will be used to restore the half-mask to its full form. This process involves the symmetrical extension of the known half-mask's geometric features to recreate the complete shape [8].

Steps to solve work Two:

(1) Symmetry Recovery: Use symmetry recovery techniques to restore the half-mask to its complete form based on the geometric features of the known half.

(2) Coordinate System: Establish a rectangular coordinate system with the tip of the nose of the golden mask as the origin.

(3) Geometric Analysis: Analyze the geometric structure of the mask by calculating the curve equations for the double eye edges and double ear edges. Determine the shapes and positions of these edges.

(4) Material Composition: Identify the composition of the golden mask, typically pure gold or gold with small amounts of alloy elements.

(5) Surface Area and Mass Calculation: Estimate the surface area and mass of the complete mask using the derived geometric data and the material composition.

By following these steps, the geometric structure and symbolic significance of the bronze sun wheel, as well as the manufacturing process, design concepts, and material properties of the golden mask are revealed. These analyses will contribute valuable data for archaeological research and cultural heritage preservation.

2.2 Model assumptions

(1) It is assumed that the other half of the golden mask is completely symmetrical to the existing half. This means that the geometric features, edge curves, and thickness are perfectly symmetrical along the axis of symmetry.

(2) It is assumed that the surface of the mask is smooth, without any protrusions, indentations, or other complex geometric features. This simplification helps in calculating the surface area.

(3) It is assumed that the material of the golden mask is uniformly distributed, meaning the density is consistent throughout the mask. This allows for accurate mass calculation by using volume and density.

(4) It is assumed that the curve equations for the double eye edges and double ear edges are continuous and differentiable. This assumption simplifies the calculations involved in surface integration.

(5) It is assumed that the thickness of the mask is constant across all areas. This assumption simplifies volume calculation by multiplying the surface area by the thickness.

3 MODEL ESTABLISHING AND ANALYZING

3.1 Model establishment and solution for work one

3.1.1 Calculation of the Five Internal Rays

A bronze wheel with a diameter of 85 cm was discovered at the Sanxingdui site. This bronze wheel is believed to be a sun symbol. The work requires establishing a mathematical model to calculate the curve equation of the bronze sun wheel and the arc equations corresponding to its five internal "rays". Needing to derive a general equation for the internal arcs and, based on this general equation, draw bronze sun wheel patterns with four, six, eight, and twelve rays [9].

(1) Basic Circle Equation of the Bronze Sun Wheel

Step 1: Determine basic parameters:

Given that the diameter of the bronze wheel is 85 cm, its radius R is 42.5 cm.

Step 2: Establish the basic circle equation:

Establishing a rectangular coordinate system with the center of the bronze sun wheel as the origin, the circle representing the bronze sun wheel can be expressed as:

$$x^2 + y^2 = R^2 \tag{1}$$

Where R is 42.5 cm.

(2) Equations of the five internal rays

Assume that each ray is a parameterized arc that can be described using parametric equations.

Step 1: Distribute the five rays uniformly:

Assuming the five rays are uniformly distributed, the angle interval for each arc is:

$$\frac{360}{5} = 72 \tag{2}$$

Step 2: Define the angles of each arc:

Since the arcs are radial, the starting and ending points of each arc lie on the same circle. The angle of each arc can be defined as:

$$\theta_k = k \cdot \frac{360°}{5} = k \cdot 72° \quad (k{=}0,1,2,3,4) \tag{3}$$

Step 3: Determine the Small Circle Radius:

Assume each arc is part of a smaller circle with radius r, and the center of the small circle lies on the circumference of the larger circle. The coordinates of the center of each small circle are:

$$\left(R\cos(\theta_k), R\sin(\theta_k)\right) \tag{4}$$

Equations of the Small Circles:

$$\left(x - R\cos(\theta_k)\right)^2 + \left(y - R\sin(\theta_k)\right)^2 = r^2 \tag{5}$$

Step 4: Equations for Each Ray:

By using the above expressions, the plot shown in Figure 1, which illustrates the bronze sun wheel are obtained.

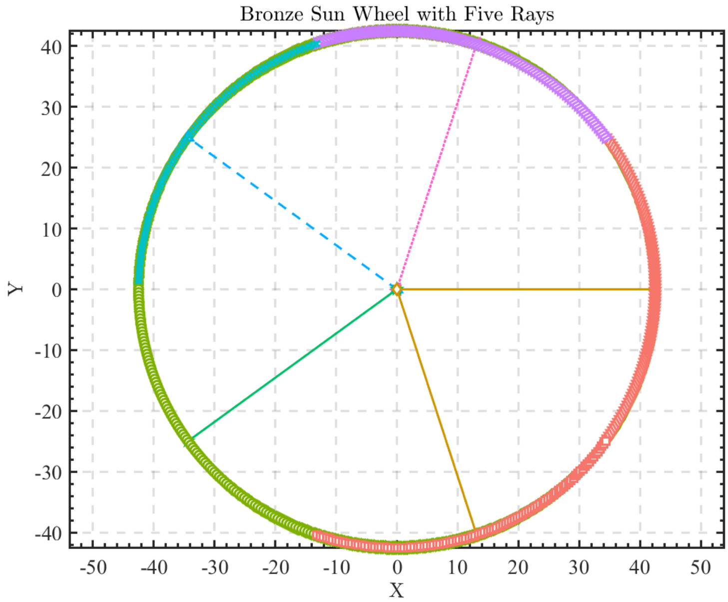

Fig.1: Bronze Sun Wheel with Five Rays

From the figure, it can be seen that the five arcs are evenly distributed within the bronze wheel, each with an angle of 72°. The starting point of each arc is the endpoint of the previous arc, and the endpoint is the starting point of the next arc [10].

3.1.2 Derivation of the General Equation

By deriving the equations for the five arcs, the general equation for the internal arcs can be obtained. First, the starting and ending angles of each arc are determined:

$$\theta_i = i \cdot 72^{\circ} \quad \theta_{i+1} = (i+1) \cdot 72^{\circ} \tag{6}$$

Where $i = 1,2,3,4, n-1$,and n is the number of rays.

Each arc can be expressed using parametric equations:

$$x(\theta) = \frac{R}{2} cos(\theta) \tag{7}$$

$$y(\theta) = \frac{R}{2} \sin(\theta) \tag{8}$$

Where, θ representing the angle change from θ_i to θ_{i+1}.

3.1.3 Drawing the Patterns

Based on the general equations of the arcs mentioned above, the patterns shown in Figures 2, 3, 4, and 5 can be obtained.

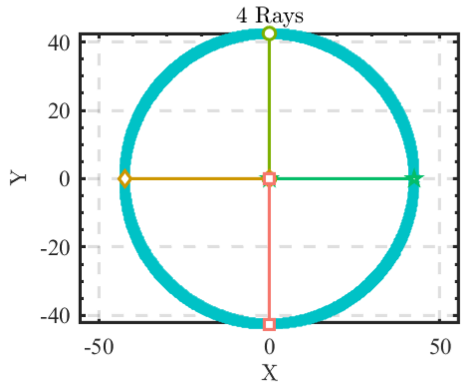

Fig.2: Four-Ray Bronze Sun Wheel Pattern

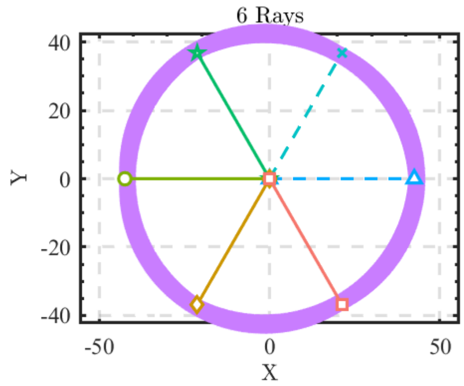

Fig.3: Six-Ray Bronze Sun Wheel Pattern

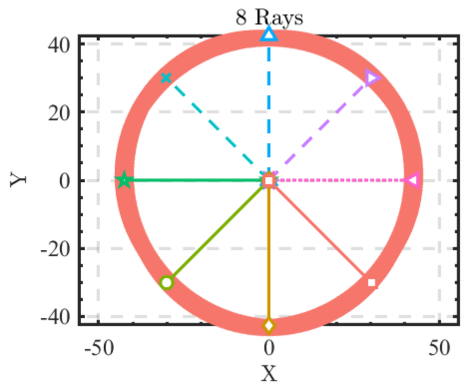

Fig.4: Eight-Ray Bronze Sun Wheel Pattern

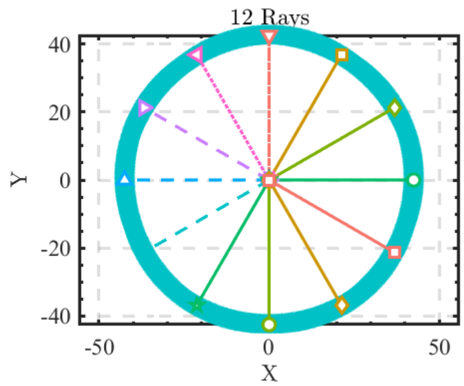

Fig.5: Twelve-Ray Bronze Sun Wheel Pattern

Due to the even distribution of the internal arc angles of the sun wheel, the resulting images shown above are uniformly distributed, with each arc being equal in size [11].

3.2 Model establishment and solution for work two

A golden mask was discovered at the Sanxingdui site. Although only half of the mask was found, its width is approximately 23 cm, and its height is approximately 28 cm. The task involves restoring this half-mask to a complete mask, establishing a coordinate system with the tip of the nose as the origin, calculating the curve equations for the "eye edges" and "ear edges," and estimating the surface area and mass of the complete golden mask [12].

3.2.1 Mask Restoration

By researching the information on the golden mask, its basic details can be obtained: the mask's thickness is 0.2 cm, and it is composed of 85% gold, 13% silver, and 2% impurities. The width is 23 cm, and the height is 28 cm. Since masks are generally symmetrical, the existing half-mask can be symmetrically extended along the vertical central axis to restore it to a complete mask [13].

Select the tip of the nose as the coordinate origin and establish a rectangular coordinate system. Assume that the tip of the nose is located at the center of the mask. Using the 3D model of the golden mask, the coordinates of the "eye edges" and "ear edges" are found. By obtaining these coordinates, appropriate curve-fitting methods are applied to derive the curve equations for the "eye edges" and "ear edges." By reading the given images and converting them to grayscale images for edge extraction and contour extraction, as shown in Figure 6.

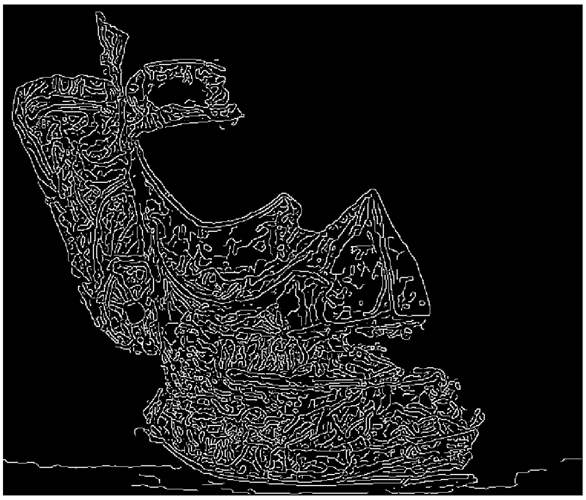

Fig.6: Edge Extraction of the Mask

By extracting the contours from the edge extraction image, the contours of the eye edges and ear edges can be filtered, and the feature point coordinates of the eye edges, ear edges, and nose tip can be obtained. Subsequently, coordinate transformation is performed to convert them into relative coordinates with the nose as the origin.

3.2.2 Polynomial Fitting

Polynomial fitting is a process of finding a polynomial function using the least squares method, such that the sum of squared errors of this polynomial function at the given data points is minimized. Polynomial fitting can be used for data smoothing, trend analysis, and prediction.

Step 1: Construct the design matrix

The design matrix X is an $n \times (m+1)$ matrix, where each row contains all polynomial terms for a data point.

$$X = \begin{bmatrix} 1 & x_1 & x_1^2 & \cdots \\ 1 & x_2 & x_2^2 & \cdots \\ \vdots & \vdots & \vdots & \cdots \\ 2 & x_n & x_n^2 & \cdots \end{bmatrix} \tag{9}$$

Step 2: Construct the target vector

The target vector y is an n-dimensional vector that contains the y-values of all the data points.

$$y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} \tag{10}$$

Step 3: Solve the normal equation

Using the least squares method, solve the normal equation to obtain the coefficient vector a of the polynomial.

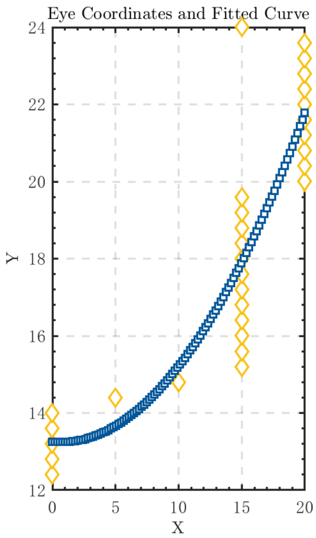

Fig.7: Fitted Curve for Eye Edge

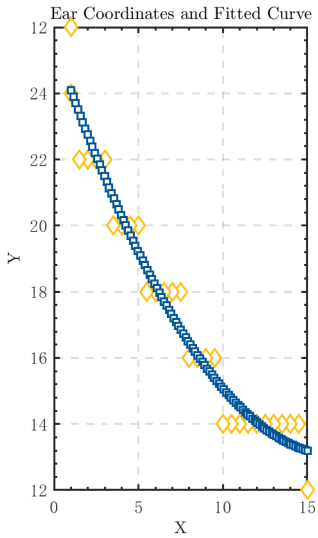

Fig.8: Fitted Curve for Ear Edge

The images above indicate that the polynomial fitting results have a low overlap with the coordinate points, leading to a poor fitting effect.

3.2.3 Spline Interpolation Fitting

Spline interpolation is a mathematical method used for fitting data points. Unlike polynomial fitting, spline interpolation uses a set of low-order polynomials to fit the data, ensuring high smoothness and accuracy at the data points. Spline interpolation is often used to approximate complex curves because it avoids the oscillation problems commonly seen in high-order polynomial fitting.

The basic idea of spline interpolation is to use low-order polynomial segments (usually cubic polynomials, i.e., cubic splines) to fit the data points, while ensuring continuity and smoothness at the data points. Cubic spline interpolation is the most commonly used spline interpolation method.

Basic Steps of spline interpolation:

Step 1: Piecewise polynomials divide the data points into multiple intervals and use a low-order polynomial (e.g., a cubic polynomial) in each interval for interpolation.

Step 2: Continuity conditions ensure that at the boundary points of adjacent intervals, the interpolation function and its first and second derivatives are continuous. This guarantees the smoothness of the interpolation curve.

Step 3: Boundary conditions apply appropriate boundary conditions at the ends of the interpolation interval (e.g., fixed endpoint slope or natural boundary conditions) to determine the specific form of the polynomial.

Mathematical Expression: Given n+1 data points, the goal of cubic spline interpolation is to construct a cubic polynomial $S(x)$ in each interval $[x_i, x_{i+1}]$ such that:

$$S(x) \quad = a_i + b(x - x_i) \quad + c(x - x_i)^2 + d(x - x_i)^3 \tag{11}$$

As shown in Figure 9, through spline interpolation fitting, the spline interpolation - fitted curve for the eye edge can be obtained; and as shown in Figure 10, the spline interpolation - fitted curve for the ear edge can be obtained.

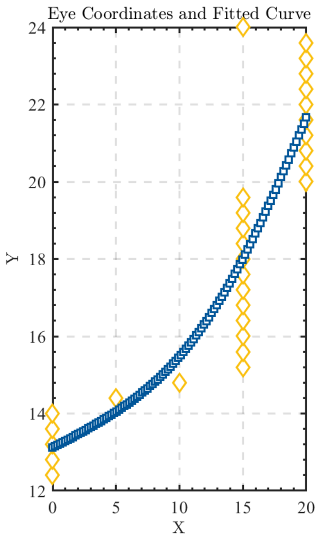

Fig.9: Spline Interpolation Fitted Curve for Eye Edge

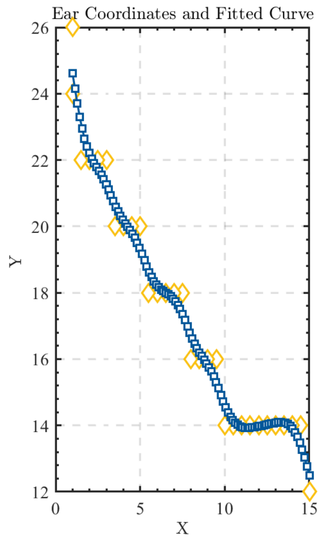

Fig.10: Spline Interpolation Fitted Curve for Ear Edge

Through the analysis of the above images, it can be seen that the spline interpolation fitted curve for the eye edge does not significantly improve compared to the polynomial fitted curve. However, the spline interpolation fitted curve for the ear edge shows a noticeable improvement. By comparing these two fitting methods, the results of the spline interpolation fitted curve are finally adopted.

3.2.4 Surface Area and Mass Estimation

By using the fitted curve equations for the ear edge and eye edge, the complete image of the mask can be reconstructed. as shown in Figure 11.

Fig.11: Complete Mask Image

To calculate the surface area and mass of the mask, surface integration based on the fitted curves is needed, along with the use of geometric and material properties for the calculation. According to the information, the mask is composed of 85% gold, 13% silver, and 2% impurities, with a thickness of 2 millimeters.

The length of the fitted curve can be calculated using the Euclidean distance formula:

$$L = \sum\sqrt{(x_{i+1} - x_i)^2 + (y_{i+1} - y_i)^2} \tag{12}$$

Surface area calculation: Assuming the mask has a uniform thickness, the surface area can be calculated using the length of the curve and the thickness.

$$S = L \times d \tag{13}$$

Where L is the length, S is the surface area, and d is the thickness.

Mass Calculation

$$V = S \times d \tag{14}$$

Where V is the volume, M is the mass, S is the surface area, the density of gold is 19.32 g/cm^3, and the density of silver is 10.49 g/cm^3. It is assumed that the density of impurities is the same as the density of silver to simplify the calculation.

$$\rho_{alloy} = 0.85 \times 19.32 + 0.13 \times 10.49 + 0.02 \times 10.49 \tag{15}$$

Therefore, the mass of the mask:

$$M^2 = V\rho_{alloy} \tag{16}$$

Based on the above process and the fitted curve equations, the surface area and mass of the mask can be calculated as 2468.432 square centimeters and 539.865 grams, respectively.

3.2.5 Model Verification

To verify the correctness of the established model, an analysis of the work is carried out. It is found that the half-mask has a width of 23 cm and a height of 28 cm. The calculated surface area of the half-mask is approximately 267 square centimeters. Consequently, when restored, the surface area of the entire mask should be around 2500 square centimeters, which closely matches the calculated result.

Furthermore, based on the available data, the mass of the half golden mask is approximately 280 grams, and that of the entire mask is around 500 grams. This is also very close to the calculated result.

4 CONCLUSIONS

(1) By precisely calculating the curve equations of the sun wheel and its rays, the model accurately describes its geometric features. The model can be adjusted according to the number of rays, making it suitable for bronze sun wheels with different numbers of rays.

(2) The sun wheel patterns drawn using programming tools can intuitively display the geometric model, facilitating analysis and understanding. The model's steps are complete, from restoring the half-mask to a full mask, to calculating the surface area and mass, covering all necessary geometric analyses.

(3) The model results can be directly used to estimate the surface area and mass of the golden mask, which is of practical significance for archaeological research and cultural heritage protection. The model combines knowledge from geometry, physics, and materials science, demonstrating strong comprehensiveness.

(4) Solving and deriving the curve equations can be complex, especially for the general equations of the internal arcs, requiring advanced mathematical knowledge and skills. Accurate geometric data are needed as input; if the data are inaccurate or incomplete, the model results may be affected.

(5) The model assumes the shapes of the sun wheel and rays are idealized, whereas actual artifacts may have errors or complex geometries. The model assumes the mask is perfectly symmetrical, while the actual artifact may have asymmetrical details, leading to deviations between the modeled mask and the real mask.

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