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Research on Sanxingdui Cultural Relic Geometry Based on Polar Coordinate Method and Linear Interpolation

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Abstract: This paper focuses on the analysis of the unearthed bronze solar wheel and gold mask. For the bronze solar wheel: Firstly, a plane coordinate system is established to transform the three dimensional plane diagram into a two dimensional plane map. Then, polar coordinates are introduced, and linear interpolation is constructed to obtain the curve equation of cultural relics. Mathematical model of the bronze solar wheel needs to be established, and the general equation of the internal arc should be derived. With the center of the bronze solar wheel as the origin of XOY, according to the general equation of the circle and by substituting the disk data, the bronze sun wheel curve equation can be obtained. On the XOY plane rectangular coordinate frame, through introducing polar coordinates, the POINT command in CAD can determine the intersection coordinates of the five rays in the disk and the angle with the horizontal axis. Substituting these into the polar coordinate equation, the five - ray equation can be obtained. To obtain the general equations of the circular arc, polar coordinates need to be established. For the drawing of bronze solar wheel four, six, eight, and 12 ray patterns, it can be obtained by adjusting the angle θ between the ray and the horizontal axis X through CAD.

Keywords: Polar coordinates; Linear interpolation; Reference point; Circular arc parameter equation; Curve equation

1 INTRODUCTION

Sanxingdui site, with a history of 3,000 to 5,000 years, is the largest ancient city, ancient town with the longest history and the richest cultural connotation in southwest China. The excavation of the Sanxingdui site is known as one of the major discoveries in the field of human archaeology in the 20th century. This discovery fully proves that the Yangtze River basin, like the Yellow River Basin, is an important birthplace of Chinese civilization and is known as the "the source of the Yangtze River civilization". The cultural relics unearthed in the site are all precious cultural heritage of human beings [1-4]. They belong to one of the cultural heritage groups with the most historical value, scientific value, cultural value, artistic value and ornamental value in China. Among the many ancient Shu treasures, a 2.62 meter bronze statue, a bronze mask with a width of 1.38 meters, and a 3.95-meter bronze sacred tree are all unparalleled treasures in the world [5].

A bronze wheel of 85 centimeters in diameter appears in shape like a contemporary steering wheel. Taking the core of the bronze sun wheel as the origin, we try to explore its

unique curve equation and the mathematical rules of the five inner "rays", so as to derive the general formula of the internal arc [6]. On this basis, based on these precise mathematical models, bronze sun wheel patterns with different layouts such as four, six, eight, and twelve rays will be drawn. It is hoped that through these patterns, the mysterious veil behind this ancient symbol will be unveiled, and the light of wisdom of Sanxingdui civilization will shine in the world again.

2 RELATED WORK AND ASSUMPTIONS

2.1 Related work

It includes establishing the general equations of its internal arc. In this process, the solar wheel is regarded as a circular disk with the central point set as the origin of the coordinate system XOY. Through CAD technology, the basic equation of the circle and the actual size parameters of the solar wheel can be derived [7]. Then, polar coordinate transformation is carried out in the same coordinate plane. The POINT command in CAD software is used to accurately locate the position of the intersection between the disk and the angle with the X - axis, which is crucial for solving the path equation of the radiation line. The equation description of the arc can be achieved by limiting the range of angle θ from 0 to 2π. Moreover, the linear interpolation method is used to convert the parametric equation to the general equation [8].

It is about drawing various ray patterns of the bronze solar wheel, such as four, six, eight and twelve ray patterns. The operation is to adjust the angle of each radiation relative to the X - axis θ, and utilize the configuration flexibility of CAD software to directly complete the graphic drawing of different ray layouts.

2.2 Model assumptions

(1) After the disk is exposed to the wind and the sun, there will be missing parts in the outer outline. In order to establish a mathematical model, the disk is assumed to be a complete disk.

(2) The isotropic hypothesis: That is, the bronze solar wheels have the same physical and mechanical properties in all directions, such as density, elastic modulus, etc.

(3) Homogeneous continuity assumption: That is, the material of the bronze solar wheel is uniform, and the properties of various materials do not change by changing the coordinate position.

(4) Suppose that the top of the five inner rays of the wheel is connected with the line segment, just forming a positive pentagon connecting the inner and the outer contour of the disk.

(5) In order to facilitate the construction of the system, all the three-dimensional masks are projected on the positive plane.

(6) Graphical assumption: the binocular sides projected to the forward plane are approximated as two arc-shaped inverted triangles, and the double ear projected to the forward plane is approximated as two arc - shaped parallelograms [9].

(7) Integrity hypothesis: assuming that the recovered half - mask is complete, that is, there are no cracks, holes or other defects inside the material.

(8) Material hypothesis: assuming that the complete half-mask is made of pure gold, that is, its physical and chemical properties are determined.

(9) Symmetry hypothesis: In order to facilitate the calculation of the binocular side and double ear curve equation, the recovered gold mask is a figure symmetric about the vertical line where the nose bridge is located.

(10) In order to facilitate the solution of the mass of the complete gold mask, it is assumed that the thickness of the complete gold mask is the same, and the complete gold mask is only made of gold without any impurities.

To clarify how to solve the work, the flow chart is shown in Figure 1.

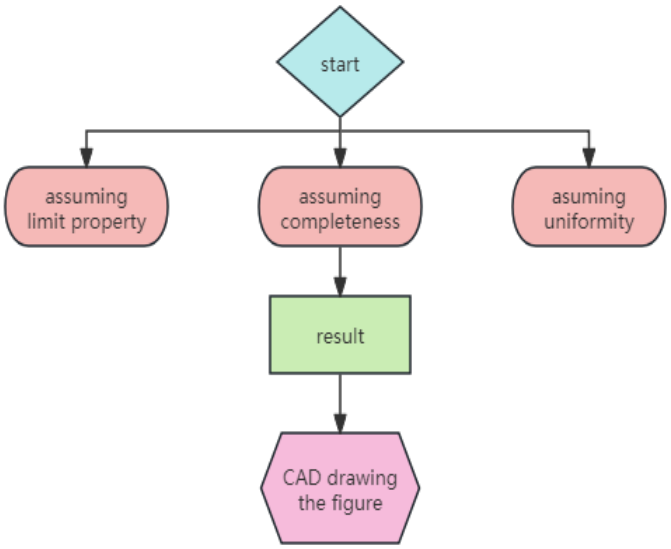

Fig. 1: The flow chart of work.

3 MODEL ESTABLISHING AND SOLVING

3.1 Preparations

According to assumption 1, the equation of the point on the disk can be written as:

$$(x-a)^2+(y-b)^2=r^2 \tag{1}$$

The bronze sun gear curve equation is:

$$x^2+y^2=1806.25 \tag{2}$$

3.2 Model establishment

Under the hypothesis, the whole bronze sun wheel can be abstracted into a circular wheel with a uniform texture. With the center of the bronze solar wheel as the coordinate origin, the plane rectangular coordinate system *XOY* is established, as shown in Figure 2.

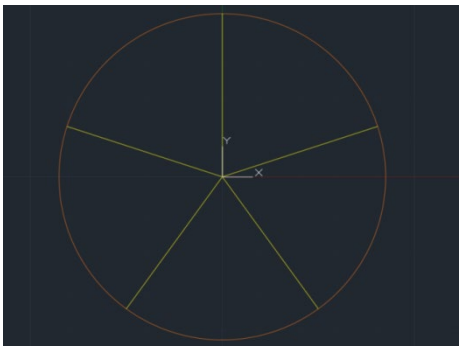

Fig. 2: Round wheel of bronze sun wheel.

This work uses CAD to re-frame the circular wheel of the bronze solar wheel on the plane to avoid some actual interference factors, and transform them from three-dimensional plane plan to further analyze the curve characteristics of the bronze solar wheel [10].

The general formula of the circles is:

$$(x-a)^2+(y-b)^2=r^2 \tag{3}$$

According to the previous model assumption and the established plane rectangular coordinate system, the coordinates of the points on the outline of the outer circle of the disk can be calculated, and the calculation formula is as follows.

$$x_i=r\cos\theta_i \tag{4}$$

$$y_i=rsin\theta_i \tag{5}$$

The general equation of the internal arc is constructed. According to the assumption (4) the internal arc of the bronze solar wheel is a circular arc. Because it is difficult to obtain the general equation of the circular arc directly in the plane right-angle coordinate system, it is necessary to construct the polar coordinates first in the solving process. To write the parameters of the arc. Its mathematical form is as follows.

$$x=a+r\cos(\theta) \tag{6}$$

$$y=a+rsin(\theta) \tag{7}$$

Where is the angle relative to the center of the circle, the range of the value determines the range of the arc. If you want to express it from the $\theta 1$ to $\theta 2$, The value changes between the two values.

If the internal arc corresponding to $L1$ and the abscissa is as follows.

$$x(\theta)=42.5\cos(\theta) \tag{8}$$

$$y(\theta)=42.5\sin(\theta) \tag{9}$$

3.3 Model solutions

(1) Curve equation of the bronze solar wheel:

(a,b) is the center coordinates of the disk, (x,y) are the coordinates of the points of the disk, and r is the radius of the disk. The diameter of the disk is 85 cm, and the center coordinate of the disk is (0,0), so the curve equation of the bronze solar wheel is:

$$x^2+y^2=1806.25 \tag{10}$$

(2) Five internal-ray equations:

On the basis of the established plane rectangular coordinate system, the polar coordinate equation can be used to calculate the coordinates of the points on the outline of the five inner rays in the outer ring of the disk. The column calculation formula is as follows:

Where, x and y are the horizontal and vertical coordinates of the points of the ith ray on the outermost circle contour of the disk, which are the angle between the ith ray and the abscissa.

(3) Derivation of the general equation of the internal arc line:

According to the previous assumption, the internal arc of the bronze solar wheel is an arc. Since it is difficult to directly obtain the general equation of the arc in the plane rectangular coordinate system, it is necessary to construct the polar coordinate to write the parameter equation of the arc first. The mathematical form is as follows:

$$x = a + r\cos(\theta) \tag{11}$$

$$y = b + r\sin(\theta) \tag{12}$$

θ is the angle relative to the corner of the circle, the $\theta's$ value range determines the range of the arc. if you want to represent a circle arc from θ1 arrive $\theta 2$, the value of θ must change between the two values.

If take $L1$ as the internal arc corresponding to the abscissa is in the polar equation form:

$$x(\theta) = 42.5\cos(\theta) \tag{13}$$

$$y(\theta) = 42.5\sin(\theta) \tag{14}$$

Take θ value range is $(0,\frac{\pi}{10})$, the arc results is shown in Figure 3.

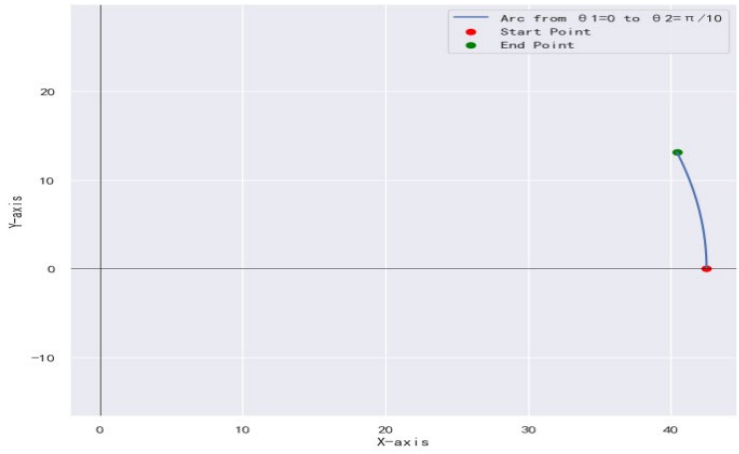

Fig. 3: Results view of the circular arc.

(4) The parametric equation is converted into a general equation:

Construct linear interpolation: Since the polar coordinate was introduced in deriving the internal arc equation, its mathematical form is the parametric equation, so the first thing to do when deriving the general equation is to transform the parametric equation into the general equation. For this, a linear interpolation is used to construct the general equation. The general form of the linear interpolation is:

$$C = A + t(B - A) \tag{15}$$

Where t is the interpolation ratio, A and B are the left and right limit values respectively.

(5) The mathematical form of the general equations:

First, the center (a, b) and radius r are determined. Second, determine the angle of the starting point and end point relative to the center θ_1 and θ_2. Then set the linear scale t to build the linear interpolation, where t is between 0 and 1. The mathematical form of the angle of the center of the circle is as follows.

$$\theta = (1 - t)\theta_1 + t\theta_2 \tag{16}$$

Then the general equation form of the circular arc is:

$$f(x, y) = [42.5cos(\theta), 42.5sin(\theta)] \tag{17}$$

(6) Drawing of the bronze sun wheel pattern of four, six, eight and twelve rays:

The CAD software is used to draw the four, six, eight and twelve-ray bronze sun wheels. The drawing results are as follows.

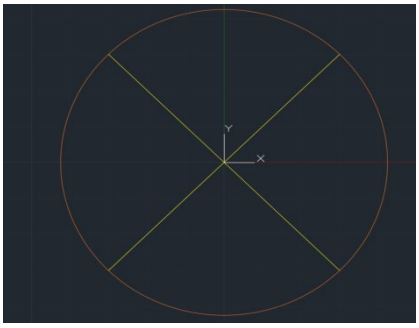

Fig. 4: A four-ray bronze solar wheel.

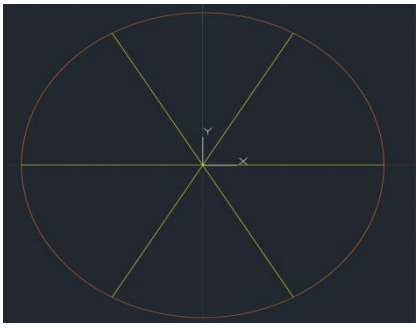

Fig. 5: Bronze Sun-wheel with six rays.

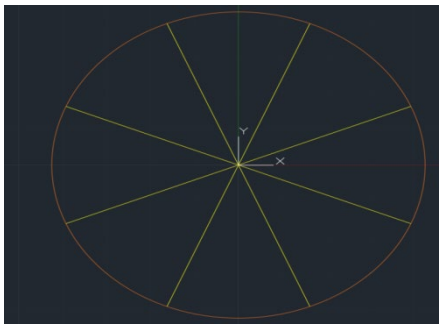

Fig. 6: A bronze solar wheel with eight-rays.

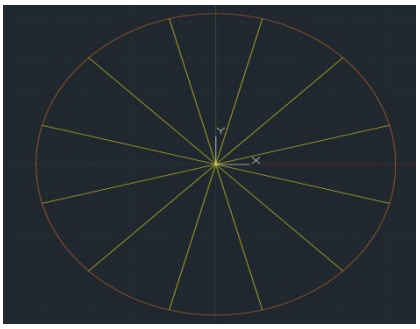

Fig. 7: Bronze solar wheel with twelve rays.

4 CONCLUSIONS

(1) Three-dimensional geometry works are analyzed by converting them into plane figures, which is a natural idea and involves a simple model.

(2) To find the curve equation of double ears and double eyes, a complex task, polar coordinate equations are introduced to establish linear interpolation. Then, by further using the flexibility of CAD functions, the coordinates of key reference points can be easily obtained. Through observing the designed graph, a satisfactory solution is achieved.

(3) The idea of material integrity and alignment is proposed. Complex works are simplified to estimate satisfactory solutions. For example, the gold mask is actually uneven, but when solving related problems, it is regarded as complete and impurity - free, thus reducing the influence of uncertain factors.

(4) The uneven surface of the gold mask brings some inconvenience to the calculation. This model can be extended to mechanical, aerospace, and civil engineering fields for the design of

complex geometries such as aircraft airfoils, bridge structures, or streamlined design of automobile bodies.

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